standard normal distribution (Z-distribution, Z)

A normalized *continuous probability distribution*. A *random variable* $Z \sim N(0, 1)$ is standard normally distributed, if it follows a *normal distribution* with mean $\mu = 0$ and variance $\sigma^2 = 1$. The *cumulative distribution function* (CDF) of the *Z*-distribution maps *z*-scores to *quantiles*. The function graphs of the *Z*-distribution are shown in figure ZDC, its *distribution parameters* and functions are listed in figure ZDD.

$Z \sim N(0,1)$	
PDF	$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$
CDF	$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{x} e^{-\frac{w^2}{2}} dw$
	$F(x) = \frac{1}{2} \left(1 + \frac{erf(x)}{\sqrt{2}} \right)$
Statistic	$x \in \mathbf{R}$: <i>z</i> -score
Parameters	none
μ	0
σ^2	1
Skewness (γ_1)	0

Figure ZDD: standard normal distribution; erf is the Gauss error function

Example: if the *z*-score of a metric, like body length, IQ, etc, is $+2.33\sigma$, then

 $F_Z(2.33) \approx 0.99$

so that score is in the 0.99^{th} quantile, i.e. it is greater than (or equal to) 99% of all scores of the population. F_Z is the CDF of the standard normal distribution. See figure ZDD.

 F_Z is often denoted using the letter ϕ , i.e. $\phi(x) = F_Z(x)$.



Figure **ZDC:** Z-distribution probability functions: left: PDF; right: CDF