geometric distribution $(X \sim Geo(p))$

A discrete probability distribution that models the probability of a given number of failures up to (and including) the first success in a fixed number of Bernoulli trials. The random variable $X \sim Geo(p)$ is geometrically distributed with a probability of success of p.

$X \sim Geo(p)$	
PMF	$f(x) = q^{x-1} \cdot p$
CDF	$F(x) = 1 - q^x$
Statistic	$x \in \mathbf{N}^+$: number of trials
Parameters	$p \in [0, 1]$: probability of success
	q: 1 - p
μ	$\frac{1}{p}$
σ^2	$\frac{q}{p^2}$
Skewness (γ_1)	$\frac{2-p}{\sqrt{q}}$

Figure **GED:** geometric distribution

For example, the geometrically distributed random variable $X \sim Geo(0.5)$ models the number of coin flips up to getting "heads" for the first time. Given that, the probability of getting heads immediately would be $f_X(1) = 0.5^0 \cdot 0.5 = 0.5$ and the probability of success on the third trial would be $f_X(3) = 0.5^2 \cdot 0.5 = 0.125$. See figure GEC for the corresponding plot.

The CDF of the geometric distribution models the case of not getting *x* failures in a row, i.e. the probability of getting success at least once in the given number of trials. E.g.: the probability of getting heads at least once in three coin flips would be $F_X(3) = 1 - 0.5^3 = 0.875$.

For a distribution modeling the probability of a number of successes given a fixed number of trials, see *binomial distribution*.



Figure **GEC:** geometric distribution probability functions; left: PMF; right: CDF; both with p = 0.5