cumulative distribution function (CDF, F_X, F)
The integral of the *probability density function* (PDF) or a sum over the *probability mass function* (PMF) of a *probability distribution*. It is denoted by the symbol F_X (or just F, if X is implied), where X is the *random variable* X or the probability distribution of X. Each distribution has its own specific CDF.

The CDF $F_X(x)$ accumulates the *probabilities* of all values in the *sample space* of X between (and including) the minimum $min(X)$ of X and its argument x, i.e. it expresses the probability $P(X \leq x)$ of X taking a value no larger than x. For *discrete* random variables, it is defined as the sum

$$F(x) = \sum_{w=min(X)}^{x} f(w)$$

and for *continuous* variables, it is the integral

$$F(x) = \int_{min(X)}^{x} f(w)dw$$

The CDF is often used to describe the probability of a random variable taking a value that falls into a specific interval in the sample space of a random variable. The probability $P(a \leq X \leq b)$ of X taking a value between (and including) a and b is

$$P(a \leq X \leq b) = F(b) - F(a)$$

The probability $F_X(x)$ of a continuous random variable X can be visualized as the area under the corresponding PDF curve from $min(X)$ to x. Its value is the proportion of the area between $min(X)$ and x and the area under the entire curve. Because the entire curve equals the sample space of X, the area under the whole PDF curve always equals $\int_{min(X)}^{max(X)} f(x)dx = 1$. Hence the value $F_X(x)$ is exactly the probability $P(X \leq x)$.

Example: the probability of a z-*score* falling within the interval $[1, 2]$ is

$$P(1 \leq Z \leq 2) = F_Z(2) - F_Z(1)$$

$$= \int_{-\infty}^{2} f_Z(x)dx - \int_{-\infty}^{1} f_Z(x)dx$$
$$\approx 0.977 - 0.841 = 0.136$$

where F_Z is the CDF of the *standard normal distribution*. See figure CDX for an illustration.

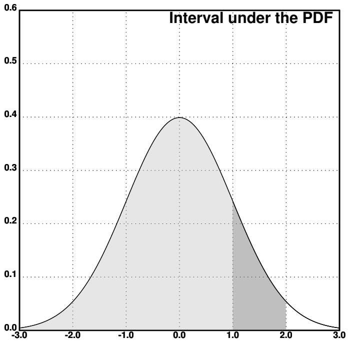

Figure **CDX:** interval under the PDF; complete gray area: $F_Z(2)$; light gray area: $F_Z(1)$; dark gray area: $F_Z(2) - F_Z(1)$; note: the domain of F_Z is $(-\infty, \infty)$, so not the entire curve is shown

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