chi-square distribution ($X \sim \chi^2(\nu)$)
A *continuous probability distribution* that models the *improbability* of an *outcome* O given an expectation E, where both O and E may be multiple *events*. The χ^2-distribution maps a χ^2 statistic (→ *chi-square statistic*) to the improbability of O arising by chance given the expectation E.

A *random variable* $X \sim \chi^2(\nu)$ is χ^2-distributed with ν (nu) *degrees of freedom*. Sometimes the notation $X \sim \chi^2_\nu$ is used instead. Often the symbol χ^2 is also used to denote a χ^2-distributed random variable or a χ^2 statistic, which can be confusing.

See figure XSD for a summary of the distribution and figure XSC for sample curves of the associated probability functions.

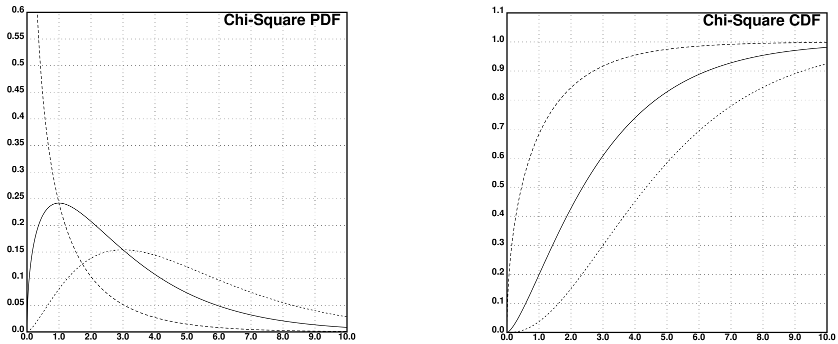

Figure **XSC:** χ^2-distribution probability functions; left: PDF of distributions with $\nu = 1$ degrees of freedom (dashed), $\nu = 3$ (solid), and $\nu = 5$ (dotted); right: CDF with the same parameters

The following example describes a test for goodness of fit. For a test for independence, see *contingency table*.

If 67 white marbles and 33 black marbles are drawn from a box given the expectation that there is a fair chance for drawing either color, the resulting χ^2 statistic would be

$$\chi^2 = \sum \frac{(67-50)^2}{50} = \frac{(67-50)^2}{50} = 5.78$$

Here the expectation would be $E(X) = 50$, because $67 + 33 = 100$ marbles are drawn in total. The statistic could also use the distance $33 - 50$ (white marbles minus expectation) instead, because the squared deviation would be the same.

The χ^2 test for goodness of fit used to find the improbability of the outcome has one degree of freedom here, because one marble count can be derived from the other. The *CDF* of the χ^2-distribution with one degree of freedom gives

$$F_{\chi^2(1)}(5.78) \approx 0.984$$

So the probability of the outcome to arise by chance given that the expectation is true is $p \approx 1 - 0.984 = 0.016$ and therefore the outcome does not fit the expectation well.

When this result would occur in the *experiment* of a *hypothesis test*, it might be significant (→ *significance*).

$X \sim \chi^2(\nu)$		
PDF	$f(x) = \frac{1}{2^{\frac{\nu}{2}} \cdot \Gamma(\frac{\nu}{2})} \cdot x^{\frac{\nu}{2}-1} \cdot e^{-\frac{x}{2}}$	
CDF	$F(x) = \frac{1}{2^{\frac{\nu}{2}} \cdot \Gamma(\frac{\nu}{2})} \cdot \int_0^x w^{\frac{\nu}{2}-1} \cdot e^{-\frac{w}{2}} dw$	
	$F(x) = P(\frac{\nu}{2}, \frac{x}{2})$	
Statistic	$x \in \mathbf{R}_0^+$: χ^2 statistic	
Parameter	$\nu \in \mathbf{N}^+$: degrees of freedom	
μ	ν	
σ^2	2ν	
Skewness (γ_1)	$\sqrt{\frac{8}{\nu}}$	
Approximation	$\sqrt{2\chi^2} - \sqrt{2\nu - 1} \sim N(0, 1)$ for $\nu \geq 30$	

Figure **XSD:** χ^2-distribution; $P(a, x)$ is the regularized incomplete Γ function, a common method for computing the CDF of the distribution