Poisson distribution $(X \sim Poi(\lambda))$

A *discrete probability distribution* modeling the *probability* of an *event* occurring x times per interval, given an average of λ events per interval, where an "interval" may be temporal, spatial, or abstract. The average number of events per interval must be constant. A *random variable* $X \sim Poi(\lambda)$ is Poisson-distributed with an average number of λ events per interval.

For example, the number of water drops falling from a leaky pipe with an average of ten drops per minute follows the Poisson distribution $X \sim Poi(10)$. Then the probability of 15 drops falling from the pipe in one minute is

$$f_X(15) = \frac{10^{15} \cdot e^{-10}}{15!} \approx 0.035$$

and the probability of up to 15 drops falling from the pipe is:

$$F_X(15) = \sum_{i=0}^{i} \frac{10^i \cdot e^{-10}}{i!} \approx 0.92$$

See figure POD for distribution parameters and figure POC for sample plots of the distribution functions.

$X \sim Poi(\lambda)$	
PMF	$f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$
CDF	$F(x) = \sum_{i=0}^{x} \frac{\lambda^i \cdot e^{-\lambda}}{i!}$
	$F(x) = Q(x+1, \lambda)$
Statistic	$x \in \mathbf{N}_0$: number of events
Parameters	$\lambda \in \mathbf{N}_0$: events per interval
μ	λ
σ^2	λ
Skewness (γ_1)	$\frac{1}{\sqrt{\lambda}}$
Approximations	$N(\lambda, \lambda)$ for $\lambda \geq 15$

Figure **POD:** Poisson distribution; Q is the complement of the regularized incomplete Γ function, a common way for computing the CDF of the distribution

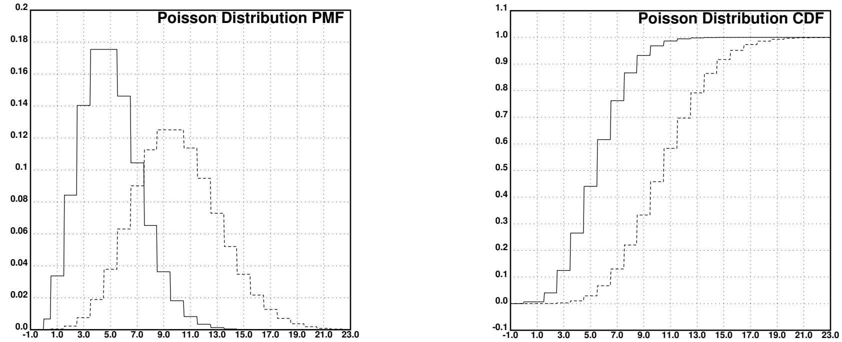

Figure **POC:** Poisson distribution probability functions; left: PMF with $\lambda = 5$ (solid) and $\lambda = 10$ (dashed); right: CDF with same parameters