t-test for location

A *hypothesis test* used to find out whether a *sample mean* $\bar{x}$ differs significantly ($\rightarrow$ *significance*) from a known mean μ_0. It uses the t*-distribution* to compute the improbability of the *observation* $\bar{x}$. The t-test for location is used when the sample size n is small, typically $n < 30$, and the population is approximately normally distributed. For larger sample sizes, the z-test can be used instead.

The *score* for the test and the test procedure are the same as for the z*-test for location*, but a t-distribution with ν (nu) *degrees of freedom* is used instead of the *standard normal distribution* to compute the *quantile* of the score. The value of ν is $n - 1$, the sample size minus one.

Example: A new technology is supposed to decrease the fault rate of a machine. The original fault rate is μ_0 = 120 failures per year, and a sample of ten new machines showed an average fault rate of $\bar{x} = 104$ with a variance of $s^2 = 360$. The *null hypothesis* H_0 is $\bar{x} \geq \mu_0$ (the new technology has no effect) and the chosen *level of significance* is $\alpha = 0.05$. Then the t*-score* is

$$t = \frac{\bar{x} - \mu_0}{\sqrt{s^2/n}} = \frac{104 - 120}{\sqrt{360/10}} = \frac{-16}{6} \approx -2.667$$

and follows a t-distribution $T_9 \sim t(9)$. The quantile associated with the t-score is

$$F_{t(9)}(-2.667) \approx 0.987$$

where $F_{t(9)}$ is the *cumulative distribution function* of the t-distribution with nine degrees of freedom. Due to the chosen null hypothesis, this is a *one-tailed* test and the result falls within the *critical region* $P(T_9 \leq t) = \alpha$, so H_0 can be rejected. See z*-test for location* for further details.